Name:____Date_____________

Integration by Substitution

Period 2

Overview: In this lesson, students will continue to practice substitution and review the chain rule and use that knowledge to learn the method of substitution for integration. You will begin with very simple examples that can be solved using the guess-and-check method and then slowly move into more complicated examples in order to help you learn how to choose the best substitutions.

Grade Level/Subject: This lesson is for 12th graders in Calculus.

Purpose: This lesson will enable you to perform more complicated integrations in a much simpler manner than guessing and checking. It will also give you practice in recognizing the common trig integrations and in recognizing derivatives.

Prerequisite Knowledge:

Students should:

- Already know and understand derivatives and know the derivatives of all trig functions.
- Know and be able to use basic integration techniques

Objectives:

1. Students will be able to use substitution as a method of solving indefinite and definite integrals.
2. Students will be able to use the substitution technique to help in recognizing a derivative produced by the chain rule.

Resources/Materials Needed:

1. Calculus Book

Activities and Procedures:

1. Chain Rule Review: Find the Derivative of the following:
    a. $\sin(x^3)$

    b. $e^{(t\wedge 2+1)}$

    c. $(x^2+1)^2$

What method did you use to find these derivatives?
Using the derivatives that you just found to find the following integrals:

a. $\int\cos(x^3)(3x^2)dx$

b. $\int e^{(t\wedge 2+1)}(2t)dt$

c. $\int 4x(x^2+1)dx$

How did you know these?

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The guess and check method worked for these, but you aren't always going to do more complex integrals this easily.
Today we are going to learn Integration by substitution, a method that will help to simplify things as we get into more complex integrals.
Substitution involves a change of variable in order to simplify the integral. Typically the variable used is u.
We choose $u = g(x)$
$du = g^1(x)dx$

Let's look again at the previous examples.

$\int \cos(x^3)(3x^2)dx$

Do you see any relationship between the two x-terms in this integral?
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Using the substitution method, we can set $u = x^3$
$du = 3x^2$
Now, substituting these in we have:

$\int \cos(u)du$
$= \sin(u) + C$
$= \sin(x^3) + C$

But how do you know what to set u equal to?
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There are no concrete rules for this, but there are some general suggestions for choosing substitutions.
Good choices for substitutions typically are:
1) g(x) is raised to a power
2) g(x) appears in the denominator
3) g(x) appears as an "inside function" of a composition

<u>Task 1: Do these examples in pairs</u>
Examples:
a. $\int e^{(t\wedge 2+1)}(2t)dt$

b. $\int 4x(x^2+1)dx$ (change of constant in this example)

c. $\int (e^{\cos x}\sin x)dx$

d. $\int (x/\sqrt{(2x+3)}\ )dx$ (finding x in terms of u in this example)

e. $\int (x/(x+21))dx$

f. $\int \csc^3(x)\cot(x)dx$ [splitting up $\csc^3(x)$ into $\csc^2(x)\csc(x)$]

Task 2: Do the following 10 questions on your own

Name: ___________________

1) $\int x\cos(x^2)dx$

6) $\int x^2 \sin(4x^3+8)dx$

2) $\int \frac{x^4}{x^5-17}dx$

7) $\int 3x^2(x^3+1)^4 dx$

3) $\int \frac{x^5}{\sqrt[3]{2x^3+7}}dx$

8) $\int (1+\sin(x))^{5/2}\cos(x)dx$

4) $\int xe^{-x^2}dx$

9) $\int 4\cos(6x)dx$

5) $\int \frac{\ln(x)}{x}dx$

10) $\int x^2 e^{4x^3}dx$

Period 3
Task 3: Do exercise 4.5 questions 1 to 30 from your calculus text book