Example 4

Find y as a function of x, given that $\frac{d^2y}{dx^2} = 15x - 2$ and that when $x = 2$, $\frac{dy}{dx} = 25$ and $y = 20$.

Given $\frac{d^2y}{dx^2} = 15x - 2$

Integrating both sides of this equation with respect to x,

$$\frac{dy}{dx} = \int (15x - 2)dx$$

$$= \frac{15x^2}{2} - 2x + c$$

But $\frac{dy}{dx} = 25$ when $x = 2$,

$\therefore$ $25 = \frac{15(4)}{2} - 2(2) + c$ i.e. $c = -1$

$\therefore$ $\frac{dy}{dx} = \frac{15x^2}{2} - 2x - 1$

Integrating both sides of this equation with respect to x,

$$y = \frac{5x^3}{2} - x^2 - x + d$$

But $y = 20$ when $x = 2$, giving $d = 6$.

The required equation is $y = \frac{5x^3}{2} - x^2 - x + 6$.

Exercise 12A

1. Find an expression for y if $\frac{dy}{dx}$ is given by

(a) $3x^2$ (b) $2x$ (c) x^3 (d) $2x^4$ (e) 5

(f) $3x^5$ (g) $\sqrt{x}$ (h) $\frac{6}{x^2}$ (i) $-\frac{4}{x^3}$ (j) $\frac{1}{\sqrt{x}}$

(k) $2x^3 + 3x^2$ (l) $5x + 1$ (m) $2x + 9x^2$

(n) $5x^4 - 6x$ (o) $8x^3 - 12x^2$ (p) $x(4 - 3x)$

(q) $3x(x - 2)$ (r) $2x(x^3 - 4)$ (s) $(3x - 1)(x + 1)$

(t) $(x - 6)(x - 2)$

2. Integrate the following functions with respect to x.

(a) $8x^3$ (b) $12x$ (c) $5x^2$ (d) 7 (e) $7 - 2x$

(f) $\frac{6}{x^3}$ (g) $-\frac{12}{x^5}$ (h) $\frac{3x}{\sqrt{x}}$ (i) $\frac{5x^2}{\sqrt{x}}$ (j) $\frac{3x^4 + 6}{x^2}$

(k) $4x^3 + 3x^2 + 2x + 1$ (l) $2x^2(3 - 4x)$

(m) $x^4 + x^2 + \frac{1}{x^2} + \frac{1}{x^4}$.

3. Find

(a) $\int 12x\,dx$ (b) $\int (x^3 + x)\,dx$ (c) $\int x(x + 1)\,dx$

(d) $\int (x + 6)(x - 4)\,dx$ (e) $\int \frac{5}{x^4}\,dx$ (f) $\int \left(10x^4 + 8x^3 - \frac{6}{x^2}\right)dx$

(g) $\int \frac{x^4 + 1}{x^2}\,dx$ (h) $\int \frac{(1 - 3x)}{\sqrt{x}}\,dx$ (i) $\int \left(x + \frac{1}{x}\right)\left(x - \frac{1}{x}\right)dx$

4. The gradient of a curve at the point (x, y) on the curve is given by $6x$. If the curve passes through the point $(1, 4)$, find the equation of the curve.
5. Find the equation of the curve passing through the point $(-2, 6)$ and having gradient function $(3x^2 - 2)$.
6. The gradient of a curve at the point (x, y) on the curve is given by $2(1 - x)$ and the curve passes through the point $(-1, 5)$. Find the equation of the curve.
7. Find S as a function of t given that $\frac{dS}{dt} = 6t^2 + 12t + 1$ and when $t = -2$, $S = 5$.
8. Find V as a function of h given that $\frac{dV}{dh} = 2(7h - 2)$ and when $h = 2$, $V = 21$.
9. Find A as a function of p given that $\frac{dA}{dp} = 5 - 4p$ and when $p = 3$, $A = -2$.
10. The gradient of a curve at the point (x, y) on the curve is given by $(3x^2 + 8)$. If the curve and the line $2x - y - 1 = 0$ cut the y-axis at the same point, find the equation of the curve.
11. The gradient function of a curve is given by $(2x - 3)$ and the curve cuts the x-axis at two points: A(5, 0) and B. Find the equation of the curve and the coordinates of B.
12. The gradient of a curve at the point (x, y) on the curve is given by $(2x - 4)$. If the minimum value of y is 3, find the equation of the curve.
13. Find y as a function of x given that $\frac{d^2y}{dx^2} = 4 - 6x$ and that when $x = 2$, $\frac{dy}{dx} = -4$ and $y = 7$.
14. Find y as a function of x given that $\frac{d^2y}{dx^2} = 6x - 4$, $y = 4$ when $x = 1$ and $y = 2$ when $x = -1$.
15. Find y as a function of x given that $\frac{d^2y}{dx^2} = 30x$, $y = 32$ when $x = 2$ and $y = 5$ when $x = -1$.